Lesson 1 – Writing Equations Using Symbols

Essential Questions:

Discussion:

The number 1,157 is the sum of the squares of two consecutive odd integers divided by the difference between the two consecutive odd integers. Which do you prefer, the word description or the numerical description? Why?	$1{,}157=\frac{x^2+(x+2)^2}{(x+2)-x}$

Example 1:

We want to express the following statement using symbolic language: A whole number has the property that when the square of half the number is subtracted from five times the number, we get the number itself.

Example 2:

We want to express the following statement using symbolic language:

Paulo has a certain amount of money. If he spends \$6.00, then he has $\frac{1}{4}$ of the original amount left.

Example 3:

We want to write the following statement using symbolic language:

When a fraction of 57 is taken away from 57, what remains exceeds $\frac{2}{3}$ of 57 by 4.

Example 4:

We want to express the following statement using symbolic language:

The sum of three consecutive integers is 372.

Example 5:

We want to express the following statement using symbolic language:

The sum of three consecutive odd integers is 93.

On Your Own:

1. The sum of four consecutive even integers is –28.	
2. A number is four times larger than the square of half the number.	
3. Steven has some money. If he spends \$9.00, then he will have $\frac{3}{5}$ of the amount he started with.	
4. The sum of a number squared and three less than twice the number is 129.	
5. Miriam read a book with an unknown number of pages. The first week, she read five less than $\frac{1}{3}$ of the pages. The second week, she read 171 more pages and finished the book. Write an equation that represents the total number of pages in the book.	

Lesson 1 Summary:

Lesson 1 – Independent Practice

Write each of the following statements using symbolic language.

1. Bruce bought two books. One book costs $4.00 more than three times the other. Together, the two books cost him $72.

2. Janet is three years older than her sister Julie. Janet's brother is eight years younger than their sister Julie. The sum of all of their ages is 55 years.

3. The sum of three consecutive integers is 1,623.

4. One number is six more than another number. The sum of their squares is 90.

5. When you add 18 to $\frac{1}{4}$ of a number, you get the number itself.

6. When a fraction of 12 is taken away from 17, what remains exceeds one-third of seventeen by six.

7. The sum of two consecutive even integers divided by four is 189.5.

8. Subtract seven more than twice a number from the square of one-third of the number to get zero.

9. The sum of three consecutive integers is 42. Let x be the middle of the three integers. Transcribe the statement accordingly.

Lesson 2 – Linear and Nonlinear Expressions in x

Essential Questions:

Discussion:

What is an expression?	
What is an equation?	

The following chart contains both linear and nonlinear expressions in x. Sort them into two groups and be prepared to explain what is different about the two groups.

Identify which equations you placed in each group. Explain your reasoning for grouping the expressions.	

How can we distinguish between linear and nonlinear equations?	Linear: Nonlinear:

Example 1:

A linear expression in x is an expression where each term is either a constant or a product of a constant and x. For example, the expression $(57-x)$ is a linear expression. However, the expression $2x^2 + 9x + 5$ is not a linear expression.

Why is $2x^2 + 9x + 5$ not a linear expression in x?	

Example 2:

Let's examine the expression $4 + 3x^5$ more deeply. To begin, we want to identify the terms of the expression.

Is $4 + 3x^5$ a linear or non linear expression in x? Why or why not?	
Example 3: How many terms does the expression $7x + 9 + 6 + 3x$ have? What are they?	
Is $10x + 15$ a linear or nonlinear expression in x? Why or why not?	

Example 4: How many terms does the expression $5 + 9x \cdot 7 + 2x^9$ have? What are they?	
Is $5 + 9x \cdot 7 + 2x^9$ a linear or nonlinear expression in x? Why or why not?	
Example 5: Is $94 + x + 4x^{-6} - 2$ a linear or nonlinear expression in x? Why or why not?	
Example 6: Is the expression $x^1 + 9x - 4$ a linear expression in x ?	
What powers of x are acceptable in the definition of a linear expression in x?	

On Your Own:

Write each of the following statements in Exercises 1–12 as a mathematical expression. State whether or not the expression is linear or nonlinear. If it is nonlinear, then explain why.

1. The sum of a number and four times the number.	
2. The product of five and a number	
3. Multiply six and the reciprocal of the quotient of a number and seven.	
4. Twice a number subtracted from four times a number, added to 15.	
5. The square of the sum of six and a number	
6. The cube of a positive number divided by the square of the same positive number	
7. The sum of four consecutive numbers	

8. Four subtracted from the reciprocal of a number.	
9. Half of the product of a number multiplied by itself three times.	
10. The sum that shows how many pages Maria read if she read 45 pages of a book yesterday and $\frac{2}{3}$ of the remaining pages today.	
11. An admission fee of $10 plus an additional $2 per game.	
12. Five more than four times a number and then twice that sum.	

Lesson 2 Summary:

Lesson 2 – Independent Practice

Write each of the following statements as a mathematic expression. State whether the expression is linear or nonlinear. If it is nonlinear, then explain why.

1. A number decreased by three squared.

2. The quotient of two and a number, subtracted from seventeen.

3. The sum of thirteen and twice a number.

4. 5.2 more than the product of seven and a number.

5. The sum that represents the number of tickets sold if 35 tickets were sold Monday, half of the remaining tickets were sold on Tuesday, and 14 tickets were sold on Wednesday.

6. The product of 19 and a number, subtracted from the reciprocal of the number cubed.

7. The product of 15 and a number, and then the product multiplied by itself four times.

8. A number increased by five and then divided by two.

9. Eight times the result of subtracting three from a number.

10. The sum of twice a number and four times a number subtracted from the number squared.

11. One-third of the result of three times a number that is increased by 12.

12. Five times the sum of one-half and a number.

13. Three-fourths of a number multiplied by seven.

14. The sum of a number and negative three, multiplied by the number.

15. The square of the difference between a number and 10.

Lesson 3 – Linear Equations in x

Essential Questions:

Concept Development: We want to define a linear equation in x

Using what you know about the words **linear** and **equation** to develop a mathematical definition of a "linear equation in x"	

$x + 11 = 15$	$5 + 3 = 8$	$-\frac{1}{2}x = 22$
$15 - 4x = x + \frac{4}{5}$	$3 - (x + 2) = -12x$	$\frac{3}{4}x + 6(x - 1) = 9(2 - x)$

Consider the following equations. Which are true, and how do you know? $4 + 1 = 5$ $6 + 5 = 16$ $21 - 6 = 15$ $6 - 2 = 1$	
Is $4 + 15x = 49$ true? How do you know?	

Define linear equation in x:	
What points can we make about linear equations in x?	
Example 1: $4 + 15x = 49$ Is there a number x that makes the linear expression $4 + 15x$ equal to the linear expression 49?	
Example 2: $8x - 19 = -4 - 7x.$ Is 5 a solution to the equation?	

Example 3:

$3(x + 9) = 4x - 7 + 7x.$

Is $\frac{5}{4}$ a solution to the equation?

Example 4:

$-2x + 11 - 5x = 5 - 6x.$

Is 6 a solution to the equation?

On Your Own:

1. Is the equation a true statement when x = –3; in other words, is –3 a solution to the equation

$6x + 5 = 5x + 8 + 2x$?

Explain.

2. Does x = 12 satisfy the equation

$16 - \frac{1}{2}x = \frac{3}{4}x + 1$?

Explain.

3. Chad solved the equation $24x + 4 + 2x = 3(10x - 1)$ and is claiming that x = 2 makes the equation true. Is Chad correct? Explain.	
4. Lisa solved the equation $x + 6 = 8 + 7x$ and claimed that the solution is $x = -\frac{1}{3}$. Is she correct? Explain.	
5. Angel transformed the following equation from $6x + 4 - x = 2(x + 1)\ to\ 10 = 2(x + 1)$. He then stated that the solution to the equation is x = 4. Is he correct? Explain.	

6. Claire was able to verify that x = 3 was a solution to her teacher's linear equation, but the equation got erased from the board. What might the equation have been? Identify as many equations as you can with a solution of x = 3.	
7. Does an equation always have a solution? Could you come up with an equation that does not have a solution?	

Lesson 3 Summary:

Lesson 3 Independent Practice

1. Given that $2x + 7 = 27$ and $3x + 1 = 28$, does $2x + 7 = 3x + 1$? Explain.

2. Is –5 a solution to the equation $6x + 5 = 5x + 8 + 2x$? Explain.

3. Does x = 1.6 satisfy the equation $6 - 4x = -\frac{x}{4}$? Explain.

4. Use the linear equation $3(x + 1) = 3x + 3$ to answer parts (a)–(d).

a. Does x = 5 satisfy the equation above? Explain.

b. Is x = –8 a solution of the equation above? Explain.

c. Is $x = \frac{1}{2}$ a solution of the equation above? Explain.

d. What interesting fact about the equation $3(x + 1) = 3x + 3$ is illuminated by the answers to parts (a), (b), and (c)? Why do you think this is true?

Lesson 4: Solving a Linear Equation

EQ:

Concept Development

What does it mean to *solve* an equation?	
In some cases, some simple guess work can lead us to a solution. For example, consider the following equation: $4x + 1 = 13$ What number x would make this equation true?	
$3(4x - 9) + 10 = 15x + 2 + 7x$ Can you guess a number for x that would make this equation true?	
Is guessing an efficient strategy for problem solving?	

$4 + 1 = 7 - 2$ Is this equation true?	
Perform each of the following operations and state whether or not the equation is still true.	
Add three to both sides of the equal sign.	
Add three to the left side of the equal sign, and add two to the right side of the equal sign.	
Subtract six from both sides of the equal sign	
Subtract three from one side of the equal sign, and subtract three from the other side of the equal sign.	
Multiply both sides of the equal sign by ten.	
Multiply the left side of the equation by ten and the right side of the equation by four.	

Divide both sides of the equation by two. Divide the left side of the equation by two and the right side of the equation by five.	
What do you notice? Describe any patterns you see.	• • • •

Example 1: Solve the linear equation. $2x - 3 = 4x$	List the properties that you use
Example 2: Solve the linear equation. $\frac{3}{5}x - 21 = 15$	List the properties that you use

Example 3:

What are other properties that make solving an equation more efficient?

-
-

Solve the linear equation.

$\frac{1}{5}x + 13 + x = 1 - 9x + 22$

Explain your work

On Your Own:

1. Solve the linear equation $x + x + 2 + x + 4 + x + 6 = -28$ State the property that justifies your first step and why you chose it.	
2. Solve the linear equation $2(3x + 2) == 2x - 1 + x$ State the property that justifies your first step and why you chose it.	

3. Solve the linear equation $x - 9 = \frac{3}{5}x$ State the property that justifies your first step and why you chose it.	
4. Solve the linear equation $29 - 3x = 5x + 5.$ State the property that justifies your first step and why you chose it.	

5. Solve the linear equation $\frac{1}{3}x - 5 + 171 = x$. State the property that justifies your first step and why you chose it.	

Lesson 4 Summary:

Lesson 4 Independent Practice

For each problem, show your work and check that your solution is correct.

1. Solve the linear equation $x + 4 + 3x = 72$. State the property that justifies your first step and why you chose it.

2. Solve the linear equation $x + 3 + x - 8 + x = 55$. State the property that justifies your first step and why you chose it.

3. Solve the linear equation $\frac{1}{2}x + 10 = \frac{1}{4}x + 54$. State the property that justifies your first step and why you chose it.

4. Solve the linear equation $\frac{1}{4}x + 18 = x$. State the property that justifies your first step and why you chose it.

5. Solve the linear equation $17 - x = \frac{1}{3} \cdot 15 + 6$. State the property that justifies your first step and why you chose it.

6. Solve the linear equation $\frac{x+x+2}{4} = 189.5$. State the property that justifies your first step and why you chose it.

7. Alysha solved the linear equation $2x - 3 - 9x = 14 + x - 1$. Her work is shown below. When she checked her answer, the left side of the equation did not equal the right side. Find and explain Alysha's error, and then solve the equation correctly.

$$2x - 3 - 9x = 14 + x - 1$$

$$-6x - 3 = 13 + 2x$$

$$-6x - 3 + 3 = 13 + 3 + 2x$$

$$-6x = 16 + 2x$$

$$-6x + 2x = 16$$

$$-4x = 16$$

$$\frac{-4}{-4}x = \frac{16}{-4}$$

$$x = -4$$